Received: *February 3, 2000,* Accepted: *May 12, 2000*

Boundary states in Gepner models

Michihiro Naka and Masatoshi Nozaki

Department of Physics, Faculty of Science, University of Tokyo
Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan
E-mail: naka@hep-th.phys.s.u-tokyo.ac.jp,
nozaki@hep-th.phys.s.u-tokyo.ac.jp

Abstract: We extend the construction of the boundary states in Gepner models to the non-diagonal modular invariant theories, and derive the same supersymmetric conditions as the diagonal theories. We also investigate the relation between the microscopic charges of the boundary states and Ramond charges of the B-type D-branes on the Calabi-Yau threefolds with one Kähler modulus in the large volume limit.

Keywords: D-branes, Conformal Field Models in String Theory.

JHEP05(2000)027

Contents

1. Introduction

The theme of D-geometry [1] is an interesting approach to D-branes on the curved space. In this paper, we consider D-branes on the Calabi-Yau spaces in type-II string theory by using the boundary states. The BPS condition for the brane configurations on the Calabi-Yau spaces implies that the cycles on which D-branes wrap must be the special Lagrangian submanifolds for the middle-dimensional cycles and holomorphic for the even-dimensional cycles [2]. These cycles correspond to the A-type and the B-type boundary states [3]. It is well-known that Gepner model [4] based on the $N = 2$ minimal models describes the exactly soluble string propagation on the Calabi-Yau space at the special symmetry enhanced point in the moduli space. The Gepner point is smoothly related to the geometric Calabi-Yau phase [5]. Recknagel and Schomerus [6] first considered D-branes in Gepner models. They used the Cardy's construction [7] for the bosonic subalgebra of the $N = 2$ SCA and directly applied this to Gepner models with the diagonal modular invariants. Subsequently, the relation to the $N = 2$ black holes was discovered [8]. On the other hand, the Cardy's construction for the A-D-E modular invariant theories [9] has been established in [10, 11]. We apply this formalism to the boundary states in Gepner models in section 2. We find that the boundary states satisfy the same supersymmetric condition as the diagonal cases. Our construction includes the known Gepner models with a completely factorized partition function, classified in [12]. Then for these states, we compute the Witten index in Ramond sector which is interpreted as the geometric intersection form encoding the quantization condition [13]. We can use this in order to count the number of the massless modes for the given configuration.

Some interesting approach was found by [14] in order to relate the boundary states (in particular, the B-type states) to the brane configurations on the quintic Calabi-Yau in the large volume limit. The monodromy matrices found in [15] and

JHEP05(2000)027

the intersection form were used in order to determine the charge of the boundary states which represents the geometric brane configurations. The same procedure was performed for the two-parameter Calabi-Yau threefolds with elliptic or $K3$ fibrations [16, 17]. This approach might bring some insights on the study of D-branes on the Calabi-Yau spaces, thus in section 3 we pursue this procedure for the one-parameter Calabi-Yau threefolds by using the results in [18]. We calculate the charge for all the boundary states and find that there are D0-branes in some models as opposed to the quintic. We also compute the number of the moduli at the Gepner point for these states. We find that the value seems to be larger than the degrees of freedom for the deformation of the bundles. It is known that there are in some sense the nonperturbative modes which exist only at the Gepner point. Thus this calculation would include some modes which are unknown in the four-dimensional low energy effective theories.

After we had obtained the results of the present paper, we received a paper [19] which has some overlaps with section 3.

2. Boundary states in Gepner models

In this section, we construct the boundary states in Gepner models classified in [12]. Then we calculate the intersection form as the Witten index in Ramond sector.

2.1 Boundary states

We begin with the rational CFTs with the chiral algebra $\mathcal{A}$ ($= \mathcal{A}_L = \mathcal{A}_R$) which have a finite set $\mathcal{I}$ of the irreducible highest weight representation $\mathcal{H}_j$, for $j \in \mathcal{I}$. The Hilbert space of the bulk CFT is $\mathcal{H} = \oplus_{(j,\bar{j})\in \mathrm{Spec}} \mathcal{H}_j \oplus \bar{\mathcal{H}}_{\bar{j}}$ with the multiplicity $N_{j\bar{j}}$ of the left and right copies of $\mathcal{A}$. Associated with this chiral algebra, there exist the chiral fields $W(z)$, $\bar{W}(\bar{z})$ with the spin s_W. In general, if there is the automorphism Ω which preserves the equal time commutators of the algebra, the boundary conditions can be written as $W(z) = \Omega(\bar{W})(\bar{z})|_{z=\bar{z}}$. The action of Ω relates an irreducible representation $\mathcal{H}_j$ to another irreducible representation $\mathcal{H}_{\omega(j)}$. These boundary conditions are equivalent to the conditions on the boundary states: $(W_n - (-1)^{s_W}\Omega(\bar{W}_{-n}))|\alpha\rangle_\Omega = 0$, where W_n and $\bar{W}_{-n}$ are the generators of the left- and right-moving chiral algebra $\mathcal{A}$. In particular, $(L_n - \bar{L}_{-n})|\alpha\rangle = 0$. These conditions were solved by Ishibashi [20] for the rational CFTs. The explicit solution $|j\rangle\rangle$ for the irreducible representation $\mathcal{H}_j$ of the algebra is given by $|j\rangle\rangle = \sum_N |j,N\rangle \otimes U\Omega|\widetilde{j,N}\rangle$ where the sum is over all the descendants of $\mathcal{H}_j$ and U is an anti-unitary operator which acts only on the right-moving generators as $U\bar{W}_n U^{-1} = (-1)^{h_W}\bar{W}_n$. There may be some multiplicity N_{jj} for each Ishibashi state, but we omit it in order to avoid the notational complication. This state $|j\rangle\rangle$ couples to the Hilbert space $\mathcal{H}_j \otimes \mathcal{H}_{\omega(j)}$ and therefore is labelled by $\mathcal{E} = \{j|(j,\bar{j} = \omega(j)) \in \mathrm{Spec}\}$, where $\mathcal{E}$ is called the set of the exponents of the theory.

Diagram G	$g=k+2$	$j\in\mathrm{Exp}(G)$
A_n	$n+1$	$1,2,3,\ldots,n$
D_{2n+1}	$4n$	$1,3,5,\ldots,4n-1,2n$
D_{2n}	$4n-2$	$1,3,5,\ldots,4n-3,2n-1$
E_6	12	$1,4,5,7,8,11$
E_7	18	$1,5,7,9,11,13,17$
E_8	30	$1,7,11,13,17,19,23,29$

Table 1: The Coxeter number and the exponents for ADE Dynkin diagram.

The Cardy's construction [7] of the boundary states for the non-diagonal modular invariant SU(2) Wess-Zumino-Witten models has been discussed in [10, 11] and we briefly review it in the following (we adopt the notation of [11]). Let G be a Dynkin diagram of the A-D-E type with the Coxeter number g which is related to the level k by $g=k+2$. To each diagram, we associate an $n\times n$ adjacency matrix $G_{\alpha\beta}$ ($G_{\alpha\beta}=\#$ of the links between the node α and β), where n is the number of the nodes of its Dynkin diagram. This matrix is diagonalized in the orthonormal basis ψ_α^j which is labelled by the node α and the exponent j. The table 1 is the list of the Coxeter number and the exponents for each diagram. For the diagonal cases, Cardy found the consistent boundary states for the rational CFTs [7] with the help of the Verlinde formula $N_{ij}^k=\sum_{\ell\in\mathcal{I}}S_{i\ell}S_{j\ell}S^*_{k\ell}/S_{1\ell}$. He required that the partition function on the strip which is periodic in the time direction (topologically an annulus) with the boundary conditions α and β is equivalent, under the modular transformation $\tau\to-1/\tau$, to that on the cylinder between the boundary states $|\alpha\rangle$ and $|\beta\rangle$. For the non-diagonal cases, the boundary states and their conjugate states are given by

$$|\alpha\rangle=\sum_{j\in\mathrm{Exp}(G)}\frac{\psi_\alpha^j}{\sqrt{S_{1j}}}|j\rangle\rangle\,,\qquad \langle\beta|=\sum_{j\in\mathrm{Exp}(G)}\langle\langle j|\frac{(\psi_\beta^j)^*}{\sqrt{S_{1j}}}\,, \tag{2.1}$$

where the set $\{\alpha\}$ and $\{\beta\}$ label the boundary states which denote the nodes of a Dynkin diagram G. The inner product between the Ishibashi states is defined by $\langle\langle j'|\widetilde{q}^{\frac{1}{2}(L_0+\bar{L}_0-c/12)}|j\rangle\rangle=\delta_{jj'}\chi_j(\widetilde{q})$, where $\widetilde{q}=e^{-2\pi i/\tau}$ and χ_j is the character $\chi_j(\widetilde{q})=\mathrm{Tr}_{\mathcal{H}_j}\widetilde{q}^{L_0-c/24}$. Then the partition function on the cylinder becomes $Z_{\alpha\beta}=\langle\beta|\widetilde{q}^{\frac{1}{2}(L_0+\bar{L}_0-c/12)}|\alpha\rangle=\sum_{j\in\mathrm{Exp}(G)}\psi_\alpha^j(\psi_\beta^j)^*\frac{\chi_j(\widetilde{q})}{S_{1j}}$. On the other hand, the partition function as a periodic time evolution on the strip with the boundary conditions α and β becomes $Z_{\alpha\beta}=\sum_{i\in\mathcal{I}}n^i_{\alpha\beta}\chi_i(q)$, where $q=e^{2\pi i\tau}$ and $1\le i\le g$. After the S-transformation $\chi_j(\widetilde{q})=\sum_{i\in\mathcal{I}}S_{ji}\chi_i(q)$ (and $S=S^t$), we obtain the Cardy's equation

$$n^i_{\alpha\beta}=\sum_{j\in\mathrm{Exp}(G)}\frac{S_{ij}}{S_{1j}}\psi_\alpha^j(\psi_\beta^j)^*\,. \tag{2.2}$$

The fused adjacency matrices which are defined by $(V_i)_\alpha{}^\beta=n^i_{\alpha\beta}$ satisfy the fusion

JHEP05(2000)027

algebra $V_iV_j = \sum_{k\in\mathcal{I}} N_{ij}^k\, V_k$ with the fusion coefficients N_{ij}^k. This algebra determines the numerical values of $n_{\alpha\beta}^i$ recursively by $V_1 = I$, $V_2 = G$, $V_i = V_2V_{i-1} - V_{i-2}$ for $3 \le i \le g$ and we can see $V_g = 0$.

We now turn to Gepner models [4]. Gepner used the so-called "β method" to construct the supersymmetric type-II string compactifications to $d+2$ dimensions by using the tensor product of the r $N=2$ minimal models at the levels k_j ($j = 1,\ldots,r$) with the internal central charge $c = 12 - \frac{3}{2}d = \sum_{j=1}^r 3k_j/k_j+2$. In each $N=2$ minimal model, the primary fields are labelled by the three integers (ℓ, m, s). The standard range of (ℓ,m,s) is $0 \le \ell \le k$, $|m-s| \le \ell$, $s \in \{-1,0,1,2\}$ and $\ell+m+s \in 2\mathbb{Z}$. The representations with $s = 0,2$ belong to the NS sector, while those with $s = \pm 1$ to the Ramond sector. The conformal dimension h and the charge q of the primary fields $\Phi_{m,s}^\ell$ are given by $h_{m,s}^\ell = \frac{\ell(\ell+2)-m^2}{4(k+2)} + \frac{s^2}{8} \pmod 1$, $q_{m,s}^\ell = \frac{m}{k+2} - \frac{s}{2} \pmod 2$. The chiral primary state and the anti-chiral primary state are labelled by $(\ell, \pm\ell, 0)$ in the NS sector and related to the Ramond ground states $(\ell,\pm\ell,\pm1)$ by the spectral flow. Each minimal model at the level k is invariant under the $\mathbb{Z}_n \times \mathbb{Z}_2$ discrete symmetry group with $n = k+2$ for A_n, D_{2n+1}, E_6 and $n = (k+2)/2$ for D_{2n}, E_7, E_8 [12]. Their actions are $g\Phi_{m,s}^\ell = e^{2\pi i \frac{m}{n}}\Phi_{m,s}^\ell$, $h\Phi_{m,s}^\ell = e^{2\pi i \frac{s}{2}}\Phi_{m,s}^\ell$. This $\mathbb{Z}_2$ symmetry is essentially the charge conjugation. Then, let us introduce the vectors $\lambda = (\ell_1,\ldots,\ell_r)$ and $\mu = (s_0; m_1,\ldots,m_r; s_1,\ldots,s_r)$ and the $2r+1$ dimensional vectors $\beta_0 = (1;1,\ldots,1;1,\ldots,1)$, and $\beta_j = (2;0,\ldots,0;0,\ldots,0,2,0,\ldots,0)$ for $j = 1,\ldots,r$ which has 2 in the first and $(r+1+j)$-th entries and is zero everywhere else. We use $s_0 \in \{-1,0,1,2\}$ in order to characterize the irreducible (s,o,c,v) representations of the $\mathrm{SO}(d)$ current algebra generated by the d space-time fermions. Then we define the inner product among the $2r+1$ dimensional vectors as $\mu \bullet \widetilde{\mu} = -\frac{d}{8}s_0\widetilde{s}_0 - \sum_{j=1}^r \frac{s_j\widetilde{s}_j}{4} + \sum_{j=1}^r \frac{m_j\widetilde{m}_j}{2(k_j+2)}$. The β-constraints are realized as $q_{\mathrm{tot}} = 2\beta_0 \bullet \mu \in 2\mathbb{Z}+1$, $\beta_j \bullet \mu \in \mathbb{Z}$. These are consistent only for $d = 2, 6$. For $d = 4$, we have to replace d with $d+2$ in the inner product in order to impose the consistent conditions. Note that the massless fields satisfy $2\beta_0 \bullet \mu = \pm 1$. Using all the ingredients, we obtain the partition function in Gepner models describing a superstring compactified to $d+2$ dimensions as follows:

$$Z = \frac{1}{2}\frac{(\mathrm{Im}\tau)^{-d}}{|\eta(q)|^{2d}} \sum_{b_0=0}^{2K-1} \sum_{b_1,\ldots,b_r=0,1} \sum_{\lambda,\mu}^{\mathrm{ev},\beta} (-1)^{b_0} M_{\lambda\lambda'} \chi_\mu^\lambda(q) \chi_{\mu+b_0\beta_0+\sum_{j=1}^r b_j\beta_j}^{\lambda'}(\bar{q})\,, \tag{2.3}$$

where $K = \mathrm{lcm}(2, k_j+2)$ and the character is $\chi_\mu^\lambda(q) = \chi_{s_0}(q)\chi_{m_1s_1}^{\ell_1}(q)\ldots\chi_{m_rs_r}^{\ell_r}(q)$. We denote by $M_{\lambda\lambda'}$ the products of the Cappelli-Itzykson-Zuber matrices [9]. The summation $\sum^{\mathrm{ev},\beta}$ is taken that the indices satisfy the β-constraints and $\ell_j+m_j+s_j \in 2\mathbb{Z}$. We can check that eq. (2.3) is invariant under the S-transformation:

$$\begin{aligned} S_{s_0,s_0'}^{\mathrm{f}} &= \frac{1}{2}\, e^{-i\pi\frac{d}{2}\frac{s_0s_0'}{2}}\,, \\ S_{(\ell,m,s),(\ell',m',s')}^{k} &= \frac{1}{\sqrt{2}(k+2)}\ \sin\pi\frac{(\ell+1)(\ell'+1)}{k+2}\, e^{i\pi\frac{mm'}{k+2}}\, e^{-i\pi\frac{ss'}{2}}\,. \end{aligned} \tag{2.4}$$

Note that eq. (2.4) is correct only for $d = 2, 6$ and we have to replace d with $d+2$ in the case of $d = 4$. For the D_{2n}, E_7, and E_8 case, the field identification exist independently for the holomorphic and the anti-holomorphic part, i.e. $\Phi^{\ell;\bar{\ell}}_{m,s;\bar{m},\bar{s}} = \Phi^{k-\ell;\bar{\ell}}_{m+k+2,s+2;\bar{m},\bar{s}}$ and this amounts to the additional factor 1/2 in front of the partition function [12]. In the D_{2n} case, this factor is canceled for the particular values $\ell = \bar{\ell} = k/2$. For all the cases, the discrete symmetry and the Hodge number for Gepner models agree with those of the geometric Calabi-Yau manifolds [12].

Then we construct the boundary states in Gepner models. In this case, the algebra $\mathcal{A}$ is generated by the $N = 2$ SCA. There are the two boundary conditions for the U(1) current J and the superconformal generators $G^{\pm}$ which preserve half of the space-time supersymmeties [3]. They are called the A-type and the B-type boundary conditions. The A-type boundary states corresponding to D-branes wrapping on the middle-dimensional cycles satisfy

$$(J_n - \bar{J}_{-n})|B\rangle = 0\,, \qquad (G_r^+ + i\eta\bar{G}_{-r}^-)|B\rangle = 0\,, \qquad (G_r^- + i\eta\bar{G}_{-r}^+)|B\rangle = 0\,, \tag{2.5}$$

and the B-type boundary states corresponding to D-branes wrapping on the even-dimensional cycles satisfy

$$(J_n + \bar{J}_{-n})|B\rangle = 0\,, \qquad (G_r^+ + i\eta\bar{G}_{-r}^+)|B\rangle = 0\,, \qquad (G_r^- + i\eta\bar{G}_{-r}^-)|B\rangle = 0\,. \tag{2.6}$$

The choice of $\eta = \pm 1$ corresponds to the choice of the spin structure. The A-type boundary states satisfy $q = \bar{q}$ and the B-type boundary states satisfy $q = -\bar{q}$. Due to the property of the anti-unitary operator U and the mirror automorphism Ω [8], the Ishibashi states $|j\rangle\rangle = \sum_N |j,N\rangle \otimes U\widetilde{|j,N\rangle}$ satisfy the B-type boundary conditions, while the states $|j\rangle\rangle = \sum_N |j,N\rangle \otimes U\Omega\widetilde{|j,N\rangle}$ satisfy the A-type boundary conditions, where the Ishibashi states are labeled by the quantum numbers of the $N = 2$ primary fields: (ℓ, m, s). One should note that they label the irreducible representations of the bosonic sub-algebra of the $N = 2$ algebras rather than the full $N = 2$ representations. Before we describe the boundary states in Gepner models, it is convenient to change the notation of ℓ into $\ell + 1 \in \mathrm{Exp}(G)$ in order to fit the range of ℓ to start from zero. Then let $|\lambda,\mu\rangle\rangle_A$ ($|\lambda,\mu\rangle\rangle_B$) be the tensor products of r A-type (B-type) Ishibashi states and the external part. Following the procedure of [6], we write down the boundary states in Gepner models as

$$|\alpha\rangle\rangle_\Omega \equiv |S_0; (L_j, M_j, S_j)_{j=1}^r\rangle_\Omega = \frac{1}{\kappa_\alpha^\Omega} \sum_{\lambda+1\in\mathrm{Exp}(G),\mu}^{\beta} \delta_\Omega\, B_\alpha^{\lambda,\mu}\, |\lambda,\mu\rangle\rangle_\Omega, \tag{2.7}$$

where S_0, L_j, M_j, S_j are integers and $\ell_j + 1 \in \mathrm{Exp}(G_j)$ for $1 \leq j \leq r$, and the normalization constant κ_α^Ω is determined later. The explicit form of $B_\alpha^{\lambda,\mu}$ is

$$B_\alpha^{\lambda,\mu} = (-1)^{\frac{s_0^2}{2}} e^{-i\pi\frac{d}{2}\frac{s_0 S_0}{2}} \prod_{j=1}^{r} \sqrt{\frac{k_j+2}{2}}\, \frac{\psi_{L_j}^{\ell_j}}{\sin^{\frac{1}{2}}\pi\frac{\ell_j+1}{k_j+2}}\, e^{i\pi\frac{m_j M_j}{k_j+2}}\, e^{-i\pi\frac{s_j S_j}{2}}\,. \tag{2.8}$$

JHEP05(2000)027

The symbol δ_Ω $(\Omega = A, B)$ denotes the constraint that the Ishibashi state $|\lambda,\mu\rangle\rangle_\Omega$ must appear in the partition function (2.3) [6]. For the A-type boundary states, this requires no constraint. However for the B-type boundary states, all the m_j are forced to be the same modulo k_j+2.

We can evaluate the partition functions on the cylinder by using eq. (2.2) and the identification $\chi^{\ell}_{m,s}(q) = \chi^{k-\ell}_{m+k+2,s+2}(q)$. The result for the A-type boundary states is

$$Z^{A}_{\alpha\widetilde{\alpha}}(q) = \frac{1}{C_A}\sum^{\mathrm{ev}}_{\lambda'\in\mathcal{I},\mu'}\sum_{\nu_0=0}^{2K-1}\sum_{\nu_1,\ldots,\nu_r=0,1}(-1)^{S_0-\widetilde{S}_0+s'_0}\,\delta^{(4)}_{S_0-\widetilde{S}_0+s'_0+\nu_0+2-2\sum_{j=1}^r\nu_j}\times$$
$$\times\prod_{j=1}^{r} n^{\ell'_j}_{L_j\widetilde{L}_j}\,\delta^{(2k_j+4)}_{M_j-\widetilde{M}_j+m'_j+\nu_0}\,\delta^{(4)}_{S_j-\widetilde{S}_j+s'_j+\nu_0-2\nu_j}\,\chi^{\lambda'}_{\mu'}(q)\,, \tag{2.9}$$

and the result for the B-type boundary states is

$$Z^{B}_{\alpha\widetilde{\alpha}}(q) = \frac{1}{C_B}\sum^{\mathrm{ev}}_{\lambda'\in\mathcal{I},\mu'}\sum_{\nu_0=0}^{2K-1}\sum_{\nu_1,\ldots,\nu_r=0,1}(-1)^{S_0-\widetilde{S}_0+s'_0}\,\delta^{(4)}_{S_0-\widetilde{S}_0+s'_0+\nu_0+2-2\sum_{j=1}^r\nu_j}\times$$
$$\times\,\delta^{(K')}_{\frac{M-\widetilde{M}}{2}+\sum_{j=1}^r\frac{K'(m'_j+\nu_0)}{2k_j+4}}\prod_{j=1}^{r} n^{\ell'_j}_{L_j\widetilde{L}_j}\,\delta^{(2)}_{M_j-\widetilde{M}_j+m'_j+\nu_0}\,\delta^{(4)}_{S_j-\widetilde{S}_j+s'_j+\nu_0-2\nu_j}\,\chi^{\lambda'}_{\mu'}(q)\,, \tag{2.10}$$

where $\mathcal{I}$ means that ℓ'_j runs from 0 to k_j; and $K' = \mathrm{lcm}(k_j+2)$, $M = \sum_{j=1}^r\frac{K'M_j}{k_j+2}$. The overall factors in eq. (2.9), (2.10) are expressed as

$$\frac{1}{C_A} = \frac{2^{\frac{r}{2}}\prod(k_j+2)}{K\kappa^A_\alpha\kappa^A_{\widetilde{\alpha}}}\,,\qquad \frac{1}{C_B} = \frac{2^{\frac{r}{2}}}{\kappa^B_\alpha\kappa^B_{\widetilde{\alpha}}}\,, \tag{2.11}$$

and we choose κ^A_α and κ^B_α to satisfy the Cardy's condition. We find that for the non-diagonal cases, the supersymmetric conditions among two boundary states take the same form

$$S_0-\widetilde{S}_0 \equiv S_j-\widetilde{S}_j \pmod 2 \qquad \text{for all } j=1,\ldots,r\,,$$
$$Q(\alpha-\widetilde{\alpha}) = -\frac{d}{2}\frac{S_0-\widetilde{S}_0}{2} - \sum_{j=1}^{r}\frac{S_j-\widetilde{S}_j}{2} + \sum_{j=1}^{r}\frac{M_j-\widetilde{M}_j}{k_j+2} \in 2\mathbb{Z}\,, \tag{2.12}$$

as the diagonal cases. This condition guarantees that there exists no tachyon in the open string spectrum.

We briefly discuss the $K3$ compactification (In this case, the different method by using the spectral flow invariant orbits was proposed in [22]). It turns out that the construction is straightforward. The difference essentially lies on $d \in 4\mathbb{Z}$. Some parts in the β-constraints and the S-modular transformation of the external part (2.4) depend on d. Thus the boundary states in Gepner models for the $K3$ can be obtained only by replacing d with $d+2$ and particularly substitute $d=4$ in the formula (2.8), (2.12). With all the things considered, we have constructed the boundary states in the Gepner models classified in [12].[1]

It is convenient to omit the space-time fermion index s_0 in order to consider only the internal CFT [14]. In this case, we adopt the ansatz for the modular invariant A-type and B-type boundary states with respect to the internal part to have the same form as eq. (2.7) but without the space-time labels s_0, S_0, i.e. $|\alpha\rangle\rangle_\Omega = |(L_j, M_j, S_j)_{j=1}^r\rangle_\Omega$. Then we set the coefficients $B_\alpha^{\lambda,\mu}$ to be

$$B_\alpha^{\lambda,\mu} = \prod_{j=1}^r \frac{1}{\sqrt{2\sqrt{2}}} \frac{\psi_{L_j}^{\ell_j}}{\sin^{\frac{1}{2}} \pi \frac{\ell_j+1}{k_j+2}} e^{i\pi \frac{m_j M_j}{k_j+2}} e^{-i\pi \frac{s_j S_j}{2}}\,, \tag{2.13}$$

and we have to set μ, β_0 and β_i to be the all $2r$-dimensional vectors without the space-time indices (the inner product should be trivially modified). Also in this case, the β-constraints mean that the internal U(1) charge is not necessarily odd-integer. Then the construction is essentially the same. The condition that the two boundary states $|\alpha\rangle\rangle$ and $|\widetilde{\alpha}\rangle\rangle$, with the same space-time part, preserve the same supersymmetries is

$$Q(\alpha - \widetilde{\alpha}) := -\frac{S-\widetilde{S}}{2} + \frac{M-\widetilde{M}}{K'} \in 2\mathbb{Z}\,, \tag{2.14}$$

where $S = \sum_{j=1}^r S_j$ and $M = \sum_{j=1}^r \frac{K' M_j}{k_j+2}$.

Before closing this subsection, we summarize the specific aspects about the diagonal cases to use in section 3. First, $\psi_{L_j}^{\ell_j}$ in eqs. (2.8), (2.13) is simply the modular S-matrix. From the Verlinde formula, $n_{L\widetilde{L}}^{\ell}$ in eqs. (2.9), (2.10) becomes $N_{L\widetilde{L}}^{\ell}$ which is the $\mathrm{SU}(2)_k$ fusion rule coefficient: $N_{L\widetilde{L}}^{\ell} = 1$ for $|L-\widetilde{L}| \le \ell \le \min\{L+\widetilde{L}, 2k-L-\widetilde{L}\}$ and $\ell + L + \widetilde{L} \in 2\mathbb{Z}$ and otherwise $N_{L\widetilde{L}}^{\ell} = 0$. Then it follows from eqs. (2.9), (2.10) that the integers (L_j, M_j, S_j) satisfy the condition $(\widetilde{L}_j - L_j) + (\widetilde{M}_j - M_j) + (\widetilde{S}_j - S_j) \in 2\mathbb{Z}$ and the identification $(L_j, M_j, S_j) \sim (k_j - L_j, M_j + k_j + 2, S_j + 2)$. Second, the boundary states have the $\mathbb{Z}_{k_j+2}$, $\mathbb{Z}_2$ symmetries $g_j : M_j \to M_j + 2$, $h_j : S_j \to S_j + 2$ inherited from the discrete symmetry in the k_j-th minimal models. Because of the β-constraints, the physically inequivalent choices for S_j are $S = 0, 2 \pmod 4$. Thus the A-type boundary states are labelled by $|\{L_j\}; \{M_j\}; S\rangle$ and their discrete symmetries are g_j's satisfying $\prod_{j=1}^r g_j = 1$. For the B-type boundary states, in addition to S_j, the physically inequivalent choices for M_j are also restricted and they are labeled by $M = \sum_{j=1}^r \frac{K' M_j}{k_j+2}$. Then each discrete symmetry g_j becomes $g_j = g^{K'/(k_j+2)}$ for $g \in \mathbb{Z}_{K'}$ and the B-type boundary states are labeled by

$$g^{M/2} h^{S/2} |\{L_i\}\rangle = |\{L_i\}; M; S\rangle. \tag{2.15}$$

We adopt the convention that (L_j, M_j, S_j) satisfy $\sum L_j + M + S \in 2\mathbb{Z}$.

[1] There are some Gepner models in type-I string theory [21]. We thank A. Sagnotti for pointing out this.

2.2 The intersection form

The CFT version of the intersection form in the classical geometry should be calculated as the Witten index $I_\Omega = \mathrm{Tr}_R(-1)^F$ in the Ramond sector in the open string channel [13]. When D-branes give rise to the particles in the macroscopic directions, this number is equivalent to the Dirac-Schwinger-Zwanziger symplectic inner product on their charges. This quantization condition was also proposed in [8].

The Witten index corresponds to the amplitude between the RR boundary states with a $(-1)^{F_L}$ inserted in the closed string channel. Only the Ramond ground states $\Phi^{\ell}_{\ell+1,1} \simeq \Phi^{k-\ell}_{-k+\ell-1,-1}$ contribute to the Witten index, thus we can evaluate it explicitly. Due to the Verlinde formula and the S-matrices, we can extend the range of the superscript ℓ of $n^{\ell}_{L\widetilde{L}}$ so that ℓ has a period of $2k+4$. We then identify $n^{-\ell-2}_{L\widetilde{L}} = -n^{\ell}_{L\widetilde{L}}$ and set $n^{-1}_{L\widetilde{L}} = n^{k+1}_{L\widetilde{L}} = 0$. In the following, we present the results for $d = 2, 6$. In order to obtain the results for $d = 4$, we have only to interchange the conditions on $d/2 + r$ as before.

The result for the A-type boundary states is given by

$$I_A = \frac{1}{\widetilde{C}_A}(-1)^{\frac{S-\tilde{S}}{2}} \sum_{\nu_0=0}^{K-1} (-1)^{(d/2+r)\nu_0} \prod_{j=1}^{r} n^{2\nu_0+M_j-\widetilde{M}_j}_{L_j\widetilde{L}_j} . \tag{2.16}$$

Then we consider the B-type boundary states. When $d/2 + r$ is even, the result is given by

$$I_B = \frac{1}{\widetilde{C}_B}(-1)^{\frac{S-\tilde{S}}{2}} \sum_{m'_j=0}^{2k_j+3} \delta^{(K')}_{\frac{M-\widetilde{M}}{2}+\sum\frac{K'(m'_j+1)}{2k_j+4}} \prod_{j=1}^{r} n^{-m'_j-1}_{L_j\widetilde{L}_j} , \tag{2.17}$$

and when $d/2 + r$ is odd, it becomes

$$I_B = \frac{1}{\widetilde{C}_B}(-1)^{\frac{S-\tilde{S}}{2}} \sum_{m'_j=0}^{2k_j+3} (-1)^{\frac{M-\widetilde{M}}{K'}+\sum\frac{(m'_j+1)}{k_j+2}} \frac{1}{2}\, \delta^{(K'/2)}_{\frac{M-\widetilde{M}}{2}+\sum\frac{K'(m'_j+1)}{2k_j+4}} \prod_{j=1}^{r} n^{-m'_j-1}_{L_j\widetilde{L}_j} . \tag{2.18}$$

We choose the normalization $\widetilde{C}_A = \kappa^A_\alpha \kappa^A_{\tilde{\alpha}} K$ and $\widetilde{C}_B = \kappa^B_\alpha \kappa^B_{\tilde{\alpha}} \prod_{j=1}^r (k_j+2)/K$ to satisfy the Cardy's condition on the partition functions.

From the above expressions, we can see that the $\mathbb{Z}_2$ action $S \to S+2$ changes the orientation of D-branes. And note that the results for B-type boundary states depend only on $M - \widetilde{M}$ expected from the discrete symmetry.

We can count the number of the moduli of these boundary states by using the intersection form. The procedure was explained in [14]. We can deal with all the *A-D-E* cases, but we deal only with the diagonal case of the B-type boundary states for the later use.

In the diagonal cases, $n^{\ell}_{L\widetilde{L}}$ becomes the $\mathrm{SU}(2)_k$ fusion coefficient $N^{\ell}_{L\widetilde{L}}$ and the intersection form can be easily rewritten in terms of the generator of the discrete symmetry. This is expressed as [16]

$$\mathbf{n}_{L\tilde{L}} = g^{\frac{|L-\tilde{L}|}{2}} + g^{\frac{|L-\tilde{L}|}{2}+1} + \cdots + g^{\frac{L+\tilde{L}}{2}} - g^{-1-\frac{|L-\tilde{L}|}{2}} - \cdots - g^{-1-\frac{L+\tilde{L}}{2}}, \tag{2.19}$$

or we can write $\mathbf{n}_{L\tilde{L}} = t_L \mathbf{n}_{00} t^t_{\tilde{L}}$ in terms of the linear transformation $t_L = t^t_L = \sum_{l=-L/2}^{L/2} g^l$ and $\mathbf{n}_{00} = (1 - g^{-1})$. When $d/2 + r$ is even, the intersection form for the B-type boundary states is given by

$$I_B = \prod_{j=1}^{r} \mathbf{n}_{L_j \tilde{L}_j} . \tag{2.20}$$

When $d/2 + r$ is odd, we can not use the formula (2.20) because the period of M is not $2K'$ but K' from eq. (2.18). Thus we have to multiply the additional factor $\frac{1}{2}(1 - g^{K'/2})$, which comes from the trivial (0-th) minimal model in order to use the above expression [17]. Then the number of the moduli is given by the diagonal part of

$$\frac{1}{2} \prod_{j=1}^{r} \tilde{\mathbf{n}}_{L_j \tilde{L}_j} P - v , \tag{2.21}$$

where $\tilde{\mathbf{n}}_{L\tilde{L}} = |\mathbf{n}_{L\tilde{L}}|$ and P is the matrix which comes from the trivial factor, namely $P = 1$ when $d/2 + r$ is even and $P = \frac{1}{2}(1 + g^{K'/2})$ when $d/2 + r$ is odd. We denote by v the number of the vacuum states. When $d/2 + r$ is odd, $v = 2^{\nu}$ where ν is the number of L_j's equal to $k_j/2$. When $d/2 + r$ is even, $v = 2^{\nu-1}$ for $\nu > 0$ and $v = 1$ for $\nu = 0$.

3. Geometric interpretation of the boundary states

The boundary states constructed in the previous section represent D-branes in the stringy scale, therefore it is interesting to relate them with D-brane configurations in the large volume limit [14]. We consider this procedure for the B-type D-branes on the one-parameter Calabi-Yau threefolds X corresponding to the $4^4 1, 6^4, 8^3 3$ Gepner models with the diagonal modular invariants which we call $k = 6, 8, 10$, respectively as in [18]. They are given by

$$\begin{aligned} W_0^{k=6} &= z_1^6 + z_2^6 + z_3^6 + z_4^6 + 2z_5^3 = 0 , \\ W_0^{k=8} &= z_1^8 + z_2^8 + z_3^8 + z_4^8 + 4z_5^2 = 0 , \\ W_0^{k=10} &= z_1^{10} + z_2^{10} + z_3^{10} + 2z_4^5 + 5z_5^2 = 0 , \end{aligned} \tag{3.1}$$

in $\mathbb{WCP}_{(\nu_1,\ldots,\nu_5)}$ where $\nu_i = k/n_i$ (n_i appear in the form $z_i^{n_i}$) and $h_{2,1} = 103, 149, 145$ for $k = 6, 8, 10$, respectively. The Kähler moduli space and the prepotential were considered in [18] following [15] where the periods of the 3-cycles on the mirror $\hat{X}$

and the mirror map were computed. In order to obtain the mirror $\hat{X}$, consider the polynomial

$$W = W_0 - k\psi z_1 z_2 z_3 z_4 z_5 = 0\,, \tag{3.2}$$

and then orbifoldize the resulting algebraic hypersurface by $G = \mathbb{Z}_3 \times \mathbb{Z}_6^2, \mathbb{Z}_8^2 \times \mathbb{Z}_2, \mathbb{Z}_{10}^2$ for $k = 6, 8, 10$, respectively [23]. The complex structure moduli space of $\hat{X}$ is a Riemann sphere with the three singularities and is parameterized by ψ^k. Basically, the BPS states wrapped on 3-cycles $Q_i[\Sigma_i]$ in $\hat{X}$ are characterized by the central charge $Z = Q_i \Pi_i$ where the periods are given by $\Pi_i = \int_{\Sigma_i} \Omega$ (Ω is the holomorphic three-form in $\hat{X}$). Each singularity in the moduli space gives the monodromy which characterizes the period.

The procedure in [14] consists of the following steps:

(i) Interpret the boundary states at the Gepner point as the generic BPS states in the moduli space.

(ii) Use the monodromy matrices in order to see the relation between the Gepner point and the large volume limit.

(iii) Translate the BPS charge vectors into the microscopic topological charges using the Chern-Simons couplings.

This procedure involves the analytic continuation between the two distinct regions in the moduli space, therefore the spectrum of the BPS states would be affected by the jumping and the marginal stability phenomena. But we are not able to treat the dynamical aspects such as the stability and the existence of bound states here. The precise form of this comparison depends on the choice of the path in the Kähler moduli space which goes from the Gepner point to the large volume limit through the complex moduli space of the mirror. This is characterized by the flat $\mathrm{Sp}(4,\mathbb{Z})$ connection provided by the special geometry.

We begin with the step (ii). At the origin ($\psi^k = 0$), Each model has an additional $\mathbb{Z}_k$ global symmetry. This is an orbifold singularity in the moduli space. The Gepner point corresponds to this point in the Kähler moduli space of X. Let A_G be the monodromy matrix induced by $\psi \to \alpha\psi$ around $\psi^k = 0$ where $\alpha = e^{2\pi i/k}$. This matrix satisfies $A_G^k = 1$. If we choose a solution $\varpi_0(\psi)$ of Picard-Fuchs equations analytic near $\psi^k = 0$, the set of solutions $\varpi_i(\psi) = \varpi_0(\alpha^i\psi)$ will provide a basis of the periods at the Gepner point

$$\varpi = -\frac{(2\pi i)^3}{\mathrm{Ord}\, G}(\varpi_2, \varpi_1, \varpi_0, \varpi_{k-1})^t\,, \tag{3.3}$$

where $\mathrm{Ord}\, G = 3{\cdot}6^2, 2{\cdot}8^2, 10^2$ with the constraints $\varpi_0+\varpi_2+\varpi_4 = 0$, $\varpi_1+\varpi_3+\varpi_5 = 0$; $\varpi_i + \varpi_{i+4} = 0 (i = 0,1,2,3)$; $\varpi_0 + \varpi_2 + \varpi_4 + \varpi_6 + \varpi_8 = 0$,[2] $\varpi_i + \varpi_{i+5} = 0$

[2] There is a typo in [18]

$(i = 0, 1, 2, 3, 4)$ for $k = 6, 8, 10$, respectively. Then let us consider the large volume limit which is mirror to the large complex structure limit ($\psi^k \to \infty$). We obtain the relation $(\gamma\psi)^{-k} \simeq e^{2\pi i(B+iJ)}$ in this limit, where $\gamma = k \prod_{i=1}^{5} \nu_i^{-\nu_i/k}$, and B is the NS B-field flux around the 2-cycle forming a basis of $H_2(X)$; and J is the size of that 2-cycle. The large volume basis is determined by the asymptotics $\psi^k \to \infty$ as in [14],

$$\Pi = \begin{pmatrix} \Pi_6 \\ \Pi_4 \\ \Pi_2 \\ \Pi_0 \end{pmatrix} = \begin{pmatrix} \mathcal{F}_2 \\ \mathcal{F}_1 \\ w_1 \\ w_2 \end{pmatrix} \simeq w_2 \begin{pmatrix} -\frac{\kappa}{6}(B+iJ)^3 \\ -\frac{\kappa}{2}(B+iJ)^2 \\ B+iJ \\ 1 \end{pmatrix}, \tag{3.4}$$

where $\kappa = 3, 2, 1$ for $k = 6, 8, 10$, respectively. The coefficients give the classical volumes of the even-dimensional cycles. The central charge corresponding to an integral vector $Q = (Q_6, Q_4, Q_2, Q_0)$ is

$$Z = Q_6\,\Pi_6 + Q_4\,\Pi_4 + Q_2\,\Pi_2 + Q_0\,\Pi_0\ . \tag{3.5}$$

We can relate ϖ to Π, i.e. $Z = Q \cdot \Pi = (Q_G M^{-1}) \cdot (M\varpi)$. This change of the basis by M is $\Pi = M\varpi$, $Q = Q_G M^{-1}$. The $\mathbb{Z}_k$ monodromy matrix in the large volume limit is given by $A = M A_G M^{-1}$. We obtain M's as follows

$$M = \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -3 & -2 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -3 & -2 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \tag{3.6}$$

for $k = 6, 8, 10$, respectively.[3] We also obtain A's as follows

$$A = \begin{pmatrix} -3 & -1 & -6 & 4 \\ -3 & 1 & 3 & 3 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -3 & -1 & -4 & 4 \\ -2 & 1 & 2 & 2 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -2 & -1 & -1 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \tag{3.7}$$

for $k = 6, 8, 10$, respectively. Note that $A^4 = -1$ for $k = 8$ and $A^5 = -1$ for $k = 10$.

Next we briefly comment on the step (iii). The BPS charge lattice of the low energy effective theory is an integral symplectic lattice which can be identified with the middle cohomology lattice of the mirror manifold $H^3(\hat{X}, \mathbb{Z})$. On the other hand, in the large volume limit, the lattice of the microscopic D-brane charges is an integral quadratic lattice identified with the K theory lattice $K(X)$. We have to construct a map between the low energy charges Q and the topological invariants of the K

[3] Of course, there is an undetermined $\mathrm{Sp}(4, \mathbb{Z})$ ambiguity. We can resolve this ambiguity by the following criteria [14]. At the conifold point ($\psi^k = 1$), the wrapped D3-brane becomes massless [24]. We choose the state which is the mirror to the conifold point to be a D6-brane with the trivial gauge bundle [25].

theory class ξ by exploiting the exact form of the Chern-Simons couplings. The effective charges of D-branes are measured by the Mukai vector $q \in H^{\mathrm{even}}(X)$ given by $q = ch(\xi)\sqrt{Td(X)}$. The central charge associated with this state is then [16]

$$Z(t) = \frac{t^3}{6}q^0 - \frac{t^2}{2}q^2 + tq^4 - q^6 \,. \tag{3.8}$$

The comparison of these central charges gives the relation between the low energy charges and the topological invariants of ξ. It is hard to see the behavior of each cycles in the case of one-parameter hypersurfaces. Thus we can not care about this mapping and have to content ourselves with determining Q.

Finally, we come to the step (i) which is the core of the method. In principle, the above central charges should be compared with those of the boundary states, but such a comparison seems to be difficult. It turns out that it is useful to compute the interaction between the two D-branes in the open string channel. This is canonically normalized because it is a partition function. Therefore we consider the intersection form. We first express the known intersection form in the large volume limit in terms of a natural basis at the Gepner point which has the $\mathbb{Z}_k$ symmetry and then compare this with that of the boundary states. Given the classical intersection form η in the large volume limit, we can determine the intersection form in the Gepner basis, $\eta_G = M^{-1}\eta(M^{-1})^t$ where η is given by $\Sigma_6 \cdot \Sigma_0 = +1$ and $\Sigma_4 \cdot \Sigma_2 = -1$ where Σ_{2n} are the even-dimensional cycles in X (The intersection form η_G is not canonically normalized, but this does not concern us.). We can write down the intersection form invariant under the $\mathbb{Z}_k$ symmetry. The $\mathbb{Z}_k$ generators act on the 3-cycles at the Gepner points $\Sigma_i^G (i = 0, \ldots, k-1)$ as $g\Sigma_i^G = \Sigma_{i+1}^G$. The results are

$$\begin{aligned} I_G^{k=6} &= g(1+g)(1-g)^3 \,, \\ I_G^{k=8} &= g(1+g)(1-g)^3(1+g^2) \,, \\ I_G^{k=10} &= g(1+g)(1-g)^2(1-g^5) \,. \end{aligned} \tag{3.9}$$

On the other hand, we obtain the intersection forms (2.20) between the $\sum L_i = 0$ states as follows

$$\begin{aligned} I_B^{k=6} &= (1-g^4)(1-g^5)^4 \,, \\ I_B^{k=8} &= (1-g^7)^4(1-g^4) \,, \\ I_B^{k=10} &= (1-g^9)^3(1-g^8)(1-g^5) \,, \end{aligned} \tag{3.10}$$

where we have inserted the trivial factors in $k = 8, 10$ as explained before. We can identify the $\mathbb{Z}_k$ generator on the both bases to obtain

$$I_B = (1-g)I_G(1-g)^t \,. \tag{3.11}$$

This equation relates the boundary states $|\{0\}; M; S\rangle$ to the basis of the periods ϖ at the Gepner point. Then the charges of the boundary states Q_B are given by $Q_G = Q_B(1-g)$.

L	Q_6	Q_4	Q_2	Q_0	moduli
00000	1	0	0	0	0
20000	-1	0	-3	0	7
11000	0	0	-3	0	8
22000	-2	0	-6	0	22
11110	-3	0	-12	0	40
22200	-4	0	-12	0	68
22110	-6	0	-18	0	94
22220	-8	0	-24	0	208

L	Q_6	Q_4	Q_2	Q_0	moduli
10000	2	1	2	-4	1
21000	-2	-1	-9	1	15
11100	0	0	-6	-3	19
22100	-4	-2	-18	2	46
21110	-6	-3	-27	3	63
22210	-8	-4	-36	4	140

Table 2: $k = 6$.

We summarize the procedure as follows. First, we take Q_G for $|\{0\};0;0\rangle$ to be $(0,1,-1,0)$ and use the monodromy matrix in order to obtain the charges in the large volume limit, i.e. $Q = Q_G M^{-1}$. Then the state $|\{0\};0;0\rangle$ corresponds to the pure six-brane $(1,0,0,0)$. We can obtain charges for different M by acting A^{-1} which implements g: $M \to M+2$. The action h: $S \to S+2$ is also implemented by $Q \to -Q$. The charges of the other states with $L = \sum_i L_i > 0$ can be obtained by $Q_G = Q_B \prod_j t_{L_j}(1-g)$. When $\sum L_i$ is odd, we also have to multiply the boundary states by $g^{1/2}$. By using the above procedure, we can determine the geometric charges for all the boundary states. We can also calculate the number of supersymmetry preserving moduli of the brane configuration which consists of the same boundary states at the Gepner point by using eq. (2.21). We find that it depends only on $\sum L_j$.

It would be meaningless to array all the results, thus we provide some interesting ones. First, the values $(\Delta M = M - \widetilde{M}, \Delta S = S - \widetilde{S})$ which satisfy the supersymmetric condition (2.14) are (0, 0) and (k, 2). Since all the k's are even integers, the configuration between the state with $\sum L_j =$ even and the state with $\sum L_j =$ odd can not be supersymmetric because of the condition $\Delta L + \Delta M + \Delta S \in 2\mathbb{Z}$. The conditions $A^4 = -1$ for $k = 8$ and $A^5 = -1$ for $k = 10$ force the independent states to satisfy only $\Delta S = \Delta M = 0$. Thus the number of independent boundary states are $90, 140, 350$ for $k = 6, 8, 10$, respectively (But we found that some different boundary states have the same charges in the large volume limit). Then we give some results on the geometric charges and the number of moduli of the boundary states in the tables 2, 3 and 4. The states $|\sum L_i \in 2\mathbb{Z};0;0\rangle$ preserving the same supersymmetry of $|\{0\};0;0\rangle$ are on the left side of the tables. The states $|\sum L_i \in 2\mathbb{Z}+1;1;0\rangle$ preserving the same supersymmetry of $|10000;1;0\rangle$ for $k = 6$ or of $|1000;1;0\rangle$ for $k = 8, 10$ are on the right side of them.

Only for $k = 6$, there are the other supersymmetric states which are not listed in table 2. On the left side, there are three independent states: $|00000;6;2\rangle$, $|11000;6;2\rangle$, $|11110;6;2\rangle$ of the charges $(-2,0,-3,0)$, $(-3,0,-6,0)$, $(-6,0,-15,0)$, which preserve the same supersymmetry of $|00000;0;0\rangle$. We can also pick up some states

L	Q_6	Q_4	Q_2	Q_0	moduli
0000	1	0	0	0	0
2000	-1	0	-2	0	6
1100	0	0	-2	0	7
3100	-2	0	-4	0	14
...					
3300	0	0	-4	0	28
...					
3333	-8	0	-32	0	496

L	Q_6	Q_4	Q_2	Q_0	moduli
1000	2	1	2	-4	3
3000	-4	-2	-8	6	6
2100	-2	-1	-6	2	11
1110	0	0	-4	-2	15
...					
3310	0	0	-8	-4	60
...					
3332	8	4	40	0	376

Table 3: $k = 8$.

L	Q_6	Q_4	Q_2	Q_0	moduli
0000	1	0	0	0	0
2000	0	0	-1	0	4
1100	1	0	-1	0	5
0001	-1	0	-1	0	3
...					
4400	0	0	-4	0	36
...					
4441	-8	0	-24	0	392

L	Q_6	Q_4	Q_2	Q_0	moduli
1000	2	1	0	-3	2
3000	-2	-1	-2	2	6
2100	0	0	-2	-1	8
1110	2	1	-2	-4	12
...					
3111	-2	-1	-8	-1	50
...					
4331	-8	-4	-24	0	316

Table 4: $k = 10$.

whose charge have the only one component except for the state $|00000; 0; 0\rangle$. For $k = 6$, there are no such states. For $k = 8$, $|3000; 2; 0\rangle$ has the charge $(0, 0, 0, -2)$. For $k = 10$, $|4000; 6; 0\rangle$, $|4200; 6; 0\rangle$, $|4440; 6; 0\rangle$ have the charges of $(0, 0, 0, -2)$, $(0, -2, 0, 0)$, $(0, -8, 0, 0)$. Thus we found D0-branes in $k = 8, 10$ as opposed to quintic. But this viewpoint may not be appropriate because we can make the linear combination of the boundary states to produce the various branes with the only one component. Thus it would be important to consider the precise map between the two bases.

We can observe the phenomena such that the non-BPS states in the large volume limit become the stable BPS states at the Gepner point. This would be related to the existence of the marginal stability lines in [1]. One should also notice that the large number of the moduli such as 496 in $k = 8$ may be related to the unknown degrees of freedom. The interpretation of this extra degrees of freedom seems to be the most urgent problem of this approach.

In these settings, the marginal operators at the Gepner point would play an important role. These boundary operator may have the superpotentials, with the flat directions corresponding to the truly marginal operators. In [14, 26], some first steps were made in order to compute the superpotentials on D-branes on Calabi-Yau.

Of course, it would be interesting to see whether the correspondence (35) is universal for the geometric interpretation of the boundary states, i.e. independent of the choice of the Gepner models. We hope to return these problems in the future work.

Acknowledgments

We are grateful to T. Eguchi and Y. Matsuo for helpful discussions. We also wish to thank J. Hashiba, M. Jinzenji, K. Sugiyama and Y. Satoh for discussions and encouragements. The work of M. Naka is supported by JSPS Research Fellowships for Young Scientists.

References

[1] M.R. Douglas, *Topics in D-geometry*, *Class. and Quant. Grav.* **17** (2000) 1057 [hep-th/9910170].

[2] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158];
M. Bershadsky, C. Vafa and V. Sadov, *D-branes and topological field theories*, *Nucl. Phys.* **B 463** (1996) 420 [hep-th/9511222].

[3] H. Ooguri, Y. Oz and Z. Yin, *D-branes on Calabi-Yau spaces and their mirrors*, *Nucl. Phys.* **B 477** (1996) 407 [hep-th/9606112].

[4] D. Gepner, *Space-time supersymmetry in compactified string theory and superconformal models*, *Nucl. Phys.* **B 296** (1988) 757.

[5] E. Witten, *Phases of $N = 2$ theories in two dimensions*, *Nucl. Phys.* **B 403** (1993) 159 [hep-th/9301042].

[6] A. Recknagel and V. Schomerus, *D-branes in Gepner models*, *Nucl. Phys.* **B 531** (1998) 185 [hep-th/9712186].

[7] J.L. Cardy, *Boundary conditions, fusion rules and the Verlinde formula*, *Nucl. Phys.* **B 324** (1989) 581.

[8] M. Gutperle and Y. Satoh, *D-branes in Gepner models and supersymmetry*, *Nucl. Phys.* **B 543** (1999) 73 [hep-th/9808080]; *D0-branes in gepner models and $N = 2$ black holes*, *Nucl. Phys.* **B 555** (1999) 477 [hep-th/9902120].

[9] A. Cappelli, C. Itzykson and J.B. Zuber, *Modular invariant partition functions in two-dimensions*, *Nucl. Phys.* **B 280** (1987) 445;
A. Kato, *Classification of modular invariant partition functions in two-dimensions*, *Mod. Phys. Lett.* **A 2** (1987) 585.

[10] G. Pradisi, A. Sagnotti and Y.S. Stanev, *The open descendants of nondiagonal* SU(2) *WZW models*, *Phys. Lett.* **B 356** (1995) 230 [hep-th/9506014].

[11] R.E. Behrend, P.A. Pearce, V.B. Petkova and J.-B. Zuber, *Boundary conditions in rational conformal field theories*, *Nucl. Phys.* **B 570** (2000) 525 [hep-th/9908036].

[12] J. Fuchs, A. Klemm, C. Scheich and M.G. Schmidt, *Spectra and symmetries of Gepner models compared to Calabi-Yau compactifications*, *Ann. Phys. (NY)* **204** (1990) 1.

[13] M.R. Douglas and B. Fiol, *D-branes and discrete torsion, 2*, hep-th/9903031.

[14] I. Brunner, M.R. Douglas, A. Lawrence and C. Romelsberger, *D-branes on the quintic*, hep-th/9906200.

[15] P. Candelas, X.C. D.L. Ossa, P.S. Green and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, *Nucl. Phys.* **B 359** (1991) 21.

[16] D.-E. Diaconescu and C. Romelsberger, *D-branes and bundles on elliptic fibrations*, *Nucl. Phys.* **B 574** (2000) 245 [hep-th/9910172].

[17] P. Kaste, W. Lerche, C.A. Lutken and J. Walcher, *D-branes on K3-fibrations*, hep-th/9912147.

[18] A. Klemm and S. Theisen, *Considerations of one modulus Calabi-Yau compactifications: Picard-Fuchs equations, Kähler potentials and mirror maps*, *Nucl. Phys.* **B 389** (1993) 153 [hep-th/9205041].

[19] E. Scheidegger, *D-branes on some one- and two-parameter Calabi-Yau hypersurfaces*, *J. High Energy Phys.* **04** (2000) 003 [hep-th/9912188].

[20] N. Ishibashi, *The boundary and crosscap states in conformal field theories*, *Mod. Phys. Lett.* **A 4** (1989) 251.

[21] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Comments on Gepner models and type-I vacua in string theory*, *Phys. Lett.* **B 387** (1996) 743 [hep-th/9607229].

[22] S. Govindarajan, T. Jayaraman and T. Sarkar, *Worldsheet approaches to D-branes on supersymmetric cycles*, hep-th/9907131.

[23] B.R. Greene and M.R. Plesser, *Duality in Calabi-Yau moduli space*, *Nucl. Phys.* **B 338** (1990) 15.

[24] A. Strominger, *Massless black holes and conifolds in string theory*, *Nucl. Phys.* **B 451** (1995) 96 [hep-th/9504090].

[25] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227];
B.R. Greene and Y. Kanter, *Small volumes in compactified string theory*, *Nucl. Phys.* **B 497** (1997) 127 [hep-th/9612181].

[26] S. Kachru, S. Katz, A. Lawrence and J. McGreevy, *Open string instantons and superpotentials*, hep-th/9912151.

JHEP05(2000)027